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double category

(Definition)

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[your settings](#)
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[user activity](#)
[user list](#)
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add to

[Encyclopædia](#)
[Papers](#)
[Books](#)
[Expositions](#)

Main Menu

sections

[Encyclopædia](#)
[Papers](#)
[Books](#)
[Expositions](#)

meta

[Requests](#) (49)
[Orphanage](#)
[Unclass'd](#)
[Unproven](#) (24)
[Corrections](#)
[Classification](#)

talkback

[Polls](#)
[Forums](#)
[Feedback](#)
[Bug Reports](#)

downloads

0.1 Background

Charles Ehresmann defined in 1963 a *double category* \mathcal{D} as an internal [category](#) in the category of [small categories](#) \mathbf{Cat} .

[Snapshots](#)
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[News](#)
[Docs](#)
[Wiki](#)
[Legalese](#)
[About](#)

0.2 Double category definition

Definition 0.1. A double category \mathcal{D} consists of:

- a set of [objects](#),

$$f : A \rightarrow B,$$

- a set of horizontal [morphisms](#)

- a set of vertical morphisms

$$j : A \rightarrow C,$$

and

- a class of [squares](#) with source and target as shown in the following [diagrams](#):

$$\begin{array}{ccc} A & \xrightarrow{f} & B \\ k \downarrow & & \downarrow g \\ C & \xrightarrow{h} & D \end{array}$$

with [compositions](#) and units of the double category that satisfy the following axioms:

- i. Horizontal:

$$A \xrightarrow{f_1} B \xrightarrow{f_2} C = [f_1, f_2] = f_2 \circ f_1$$

$$A \xrightarrow{1_A^h} A \xrightarrow{f_1} B = A \xrightarrow{f_1} B = A \xrightarrow{f_1} B \xrightarrow{1_B^h} B$$

- ii. Vertical:

$$[A \xrightarrow{j_1} B \xrightarrow{j_2} C]_{vert} = [j_1, j_2]_{vert} = j_2 \circ j_1$$

$$[A \xrightarrow{1_A^v} A \xrightarrow{j_1} B = A \xrightarrow{j_1} B = A \xrightarrow{j_1} B \xrightarrow{1_B^v} B]_{vert}.$$

Compositions for [square diagrams](#) in a double category \mathcal{D} :

- iii. Horizontal composition:

$$\begin{array}{ccc} A \xrightarrow{f_1} B & [\alpha] \circ [\beta] & B \xrightarrow{f_2} C & [\beta] = & A \xrightarrow{[f_1 f_2]} C & [\alpha \beta]. \\ j \downarrow & & k \downarrow & & j \downarrow & \downarrow l \\ D \xrightarrow{g_1} E & & E \xrightarrow{g_2} F & & D \xrightarrow{g_1 g_2} F \end{array}$$

- iv. Vertical composition of squares in \mathcal{D} : $[\alpha \beta]_{vert}$ is expressed as

$$\begin{array}{ccc} A & \xrightarrow{f} & B \\ | & & | \end{array} \quad [\alpha \beta]_v.$$

$$\begin{array}{ccc} |j_1 j_2|_v \downarrow & & \downarrow |k_1 k_2|_v \\ E & \xrightarrow{h} & F \end{array}$$

Moreover, all compositions are associative and unital, and also subject to the Interchange Law:

$$\begin{array}{ccc} [\alpha] & \xrightarrow{\quad} & [\beta] \\ \downarrow | & & \downarrow | \\ [\gamma] & \xrightarrow{\quad} & [\delta] \end{array} = [[\alpha\beta] \text{ over } [\gamma\delta]]_{\text{vert.}} = [\alpha\gamma]_v \circ [\beta\delta]_v.$$

Unit morphisms are also subject to the axioms of the double category. For further details on double categories and examples please see the related [free download PDF file](#).

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Other names: [internal category in Cat](#)

Also defines: [internal category in Cat](#), [intrchange law](#), [horizontal composition](#), [vertical composition](#), [vertical identities](#), [horizontal identities](#)

Keywords: [double category](#)

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There are [6 references](#) to this object.

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